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Can One Really Disprove a Real Quantum Theory?

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Abstract Two papers to disprove the real-number formulation of quantum theory have been published in the second issue of Physical Review Letters (2022). Based on these two papers, one may draw a solid conclusion that micro worlds are really very different from macro worlds, and it is indispensable to use complex numbers in the quantum theory. However, a philosophical assessment based on our new general system theory is that such a disproof must rely on the self-circular logic. The purpose of this paper is to explain why such a philosophical question cannot be disproved through logical reasoning and experimental methods, and any claim to prove or disprove must rely on self-circular logic, which is really the case with present disproof.

Keywords Standard quantum theory, Real quantum theory, Complex quantum theory, Disproof, Self-circular logic

1. Introduction

In the second issue of Physical Review Letters (2022), two papers [1,2] have been published to disprove the real quantum theory. From these two papers and the previous work of others cited in these two papers, one may draw a solid conclusion that the micro worlds are indeed very different from the macro worlds, and it is indispensable to use complex numbers in quantum theory, while in classical mechanics only real numbers are adequate for handling the macro worlds. This conclusion will provide a great support for the separatists and convince centrists that a unified theory covering both macro and micro worlds is impossible. According to our New General System Theory (NGST) [3,4], the question raised by the authors is fundamentally a philosophical question and its answer is a selection problem. Both yes and no are possible to construct different theories for the same world. The selection of yes or no just reflects the philosophical belief of scientists. Neither side can defeat the other by means of logical reasoning method alone. Even the physical experiments are inadequate to judge these two beliefs due to the finite nature of experiments while the assumption is often of infinite nature. Any proof or disproof of such an existential statement must rely on self-circular logic. The purpose of this paper is to explain how this conclusion is reached.

2. Historical Process of the Problem Formulation

Since the birth of classical mechanics four centuries ago [5,6], abstract mathematical entities have played an important role in formalizing physical concepts. Physicists use mathematics to describe nature. In classical physics, real numbers were early used to describe the physical reality of observed phenomena, whereas complex numbers can be employed as convenient mathematical tools. For example, complex numbers were introduced in electromagnetism to simplify calculations. People represent the electric and magnetic fields as complex vector fields to describe electromagnetic waves [7]. The role of complex number in Euclidean space is the same as a two-dimensional vector in comparing with two real numbers. Thus, very frequently two real equations can be merged into one complex equation. Hilbert space can be viewed as a generalization of Euclidean space in two aspects, one is from real numbers to complex numbers, and the other is from finite dimension to infinite dimension. The first generalization is natural while the second could change the nature of the problem. The second part has no corresponding part in Euclidean space, which could introduce many pleasant and unpleasant properties. Das specifically emphasized this problem [8] and demonstrated that Heisenberg's uncertainty principle is a consequence of Fourier transform (FT). He argued that the FT is based on the infinity assumption, which is unrealistic and meaningless both in nature and in engineering.

When von Neumann tried to establish a mathematical foundation for quantum mechanics in 1930s [9], he simply accepted the Copenhagen interpretation that micro worlds are very different from macro worlds. He thought that Euclidean geometry was inadequate to interpret the experimental phenomena of micro worlds, so he developed the mathematical quantum theory based on complex Hilbert-space formulations. This practice was followed by Dirac [10] and many others, and thus it became the standard

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quantum theory [2]. The standard quantum theory was formulated with complex-valued Schrödinger equations, wave functions, operators, and Hilbert spaces [2] while classical mechanics was traditionally formulated with real-valued governing equations. This has led some people to ask a fundamental question whether complex numbers are necessary in the standard formalism of quantum theory. This was exactly the case for the debate between real quantum theory and complex quantum theory [1,2,11-23]. Even the founder himself did not believe that the quantum theory cannot be built on a real Hilbert space [24]. This debate is similar to other debates between classical mechanics and quantum mechanics, such as deterministic versus probabilistic, locality vs causality, completeness vs hidden variables [25,26]. The essence of this debate is whether micro worlds are fundamentally different from macro worlds. More details can be found in ref [27].

3. The Problem Does Not Stand

First, from our point of view, the problem is caused by a false belief, which is actually untenable. The authors of these two papers and many others believe that "the real number appears complete to describe the physical reality in all classical phenomena" [2] and their purpose is to show that the complex number is necessary to describe the physical reality in quantum phenomena. As a matter of fact, their first belief cannot be proven to be true since all classical phenomena can never be known, so it can never be proved that the real number appears to be complete for all the classical phenomena. If that belief does not stand, then the comparative problem does not exist and so is the new problem. We cannot draw any conclusions from these two different practices to describe macro physical phenomena and micro physical phenomena using real-valued mathematical theory or complex-valued mathematical theory respectively that the micro worlds are fundamentally different from the macro worlds.

Secondly, the nature of mathematics. In classical mechanics, mathematics is a language for describing the physical states, while in quantum mechanics it has been changed to "quantum states are the key mathematical objects in quantum theory" [28]. Allen C. Dotson pointed out that in Bohr's conception of quantum states, dynamical variables that characterize a quantum state are defined in connection with specific experimental arrangements, rather than as elements of reality associated solely with the object, as Einstein would have wanted [29, p.12]. So the fundamental meanings of state and mathematics are different, and further differences can be found for other concepts such as time, space, matter, energy, etc [4]. Thus, it can be found that the problem arises from the beginning of the standard quantum theory rather than the physical reality. If we really want to compare two theories, it must first unify the fundamental concepts, otherwise it is meaningless for a comparison. If the mathematics is understood to be a language for describing the physical states in micro worlds which is defined as quantum states, then the complex-valued Schrödinger equation is just an approximation to the essence of the physical micro worlds. One can never know the essence of a world no matter it is macro or micro in scale, but we can make assumptions about it. So this is also a belief-based selection problem.

4. The Logic Behind the Disproof

Many scientists [e.g. 14-23] selected to believe that quantum systems can be universally simulated using only real numbers by exploiting an expanded Hilbert space in various alternative formalisms of quantum theory, but more scientists intend to believe that is not possible. The reason why these scientists failed to convince others is the same as Einstein failed to convince Bohr on the causality law [25-27]. In accepting the Hilbert space as a replacement for Euclidean geometry, one has already fallen into the trick since the Hilbert space is not a direct generalization of Euclidean space. It has included the special new feature of the infinite dimension. In parallel with Bell's method to falsify the existence of local hidden variable theories to describe the micro world [30-31], scientists devised a Bell-like experiment to disprove the real quantum theory [13]. This is a very trick with the belief that a physics problem is equivalent to a mathematical problem. As a matter of fact, no matter what method of logical reasoning is used, the existential statement such as "unicorns exist", or the universal statement such as "all swans are white" cannot be proven or disproved logically except by physical observations of the entire sample space. The research program of singling out quantum correlations by demanding maximal performance in a device-independent information theoretic task was initiated by Weilenmann and Colbeck in 2020 [11,12]. It was first addressed by Renou et al. in 2021 [13] and the results show that complex quantum theory outperforms real quantum theory when the non-local game T is played in the entanglement-swapping scenario. Their experimental disproof of real quantum physics based on the inequality T gives $T \lesssim 7.66$. Li et al. [1] tailored such tests for implementation in state-of-the-art photonic systems. They experimentally demonstrated quantum correlations in a network of three parties and two independent EPR sources that violate the constraints of real quantum theory by over 4.5 standard deviations. Chen et al. [2] developed a quantum game to distinguish standard quantum theory from its real number analog, by revealing a contradiction between a high-fidelity multiqubit quantum experiment and players using only real-number quantum theory. By using superconducting qubits, they realized the quantum game based on deterministic entanglement swapping with a state-of-the-art fidelity of 0.952. Their experimental results violate the real-number bound of 7.66 by 43 standard deviations. If one accepts this change of a physics problem to a mathematical problem, then the work of both Li et al. [1]

and Chen et al. [2] is very solid and this problem has been solved. They have really disproved the real quantum theory as a universal physical theory. However, all the above disproof methods are based on the acceptance of four fundamental axioms of standard quantum theory [32] and thus it is a self-circular logic. In the next section, we will reveal this trick. For those who want to be against the belief that it is indispensable to use complex numbers in quantum theory, what he needs to do are: (1) to describe the quantum states in real-valued Euclidean geometry similar as classical mechanics [e.g. 35]; (2) not to accept the belief that a physics problem is equivalent to a mathematical problem as argued in this section.

5. Problems Existed in the Disproof Process

5.1. Problems Embedded in a Language

If one wants to defend the real quantum theory, then one needs to stick to the Euclidean space. Furthermore, he needs to emphasize that it is not the language which determines the nature of the world. Fundamentally speaking, since the whole universe is a network of interacting entities, human beings can never observe the whole universe, human beings can never reveal the actual nature of the universe. However, human beings can reveal approximately the nature of a world which is a finite part of the universe in a relative sense [3,4]. Therefore, any claim of the statement such as “micro worlds are fundamentally different from macro worlds” is an overclaim and untestable with scientific methods.

In terms of the definitions of objects, since the actual number of objects in the universe can never be known, every dictionary either uses a self-circular logic to define each object if all the objects in a dictionary are defined (completeness sacrifices logical consistency) or leaves some object names undefined and treats them as understandable to everyone (logical consistency sacrifices completeness). Thus, no language can satisfy both completeness and logical consistency. So the statement that “the real number appears complete to describe the physical reality in all classical phenomena” is a false belief. But if we leave the real numbers as an open series, then any observed item can be assigned a different number, so that the real numbers are adequate to describe the physical reality no matter in micro or macro worlds. Therefore, the problem for the debate does not exist. Whether one uses two real numbers or one complex number is just a selection problem, and it can apply to any physical world.

5.2. Mathematics is a Special Language

Mathematics is a special language developed by human beings. Mathematics has lots of implicit and explicit assumptions [8]. Philosophically speaking, the truth of a universal or existential statement can only be verified or disproved by observations of all sample spaces, not by means of logical reasoning alone [33]. For example, “all swans are white” and “unicorns exist”, the truth of these statements can only be tested by observation of all sample spaces. Without observation, truth cannot be proved or disproved by any method of logical reasoning. If the axioms are partially correct, then partially correct conclusions can be derived through logical reasoning methods such as induction, deduction, and abduction. In that case, if one accepts the game, then one can derive any conclusion he wants. So Bell type no-go theorem is a mathematical skill to persuade people to accept a conclusion. It looks very strict but philosophically it has flaws. The trick is that at each step, the correctness is high enough to be falsified. For example, in the standard quantum mechanics, the following four axioms are introduced and all those authors in ref. [1,2] take them for granted [2,32]: *“(1) A pure quantum system is described by a unit complex vector in a Hilbert space. (2) The state space of a composite quantum system is the tensor product of the state spaces of the component systems. (3) The dynamics of a closed quantum system is described by a unitary operator acting on the state vector. (4) A physical observable is described by a Hermitian operator, and the measurement outcome obeys the Born rule.”* In the next section, we will show that all these axioms are not 100% correct, so the foundation for the disproof is not so solid.

5.3. Problems Existed in the Axioms of Standard Quantum Theory

From our NGST’s point of view [3,4,33], all these axioms are problematic. In axiom (1), one can never create a pure quantum system. When one has the concept of pure, it implies that the actual system is not pure. They co-exist based on the relativity of simultaneity axiom [3,4]. Each concept is defined on the basis of other concepts. The minimum division is 2 since two-valued logic is the minimum logical system for human mathematics. So at least a pair of concepts have to be used for describing something. Existence of concept A relies on the co-existence of non-A. For example, good is related to bad, man is related to woman. In axiom (2), superposition is implied in this assumption which rules out nonlinearity, and this is certainly an approximation of the real physical system. In axiom (3), the closeness is the same as the concept of pure, the actual closed system does not exist physically, and whether any closed system, such as including living creatures, can be described by a unitary operator acting on the state vector is also a question needs to be examined. The existence of dark matter and dark energy in current universe model is a counter example to the assumptions of isolated or closed system models. In axiom (4), the Hermitian operator and Born rule are also two very big assumptions, unique to the creation of standard quantum theory. If one accepts these assumptions, it implies that one has accepted the mathematical treatment of von Neumann and Dirac, then its proof of their conclusion is fundamentally a self-circular logic. If one gives up these four assumptions and start a new theory based on a set of other fundamental assumptions, a different real-valued theory is possible. The new general system theory proposed by the present author is an example [3,4,33]. Since these four assumptions are already far away from the physical reality, thus any deduction based on these assumptions are just mathematical games rather than revealing the truth of physical reality. Furthermore, the standard quantum theory is not actually a theory about the physical reality, but just a mathematical theory. If one sticks to the generalization of classical mechanics to handle the micro world phenomena, then the same is possible for the real quantum theory. However, the use of complex number in quantum theory may have some of the same conveniences as electromagnetic theory.

5.4. Some Further Supports

In the preface of the book, physicist and mathematician Lazar Mayants pointed out that "when I scrutinize thoroughly the conventional foundations of quantum physics, I found them rather unsound and came to the conclusion that to comprehend what quantum physics really is, it is necessary, in the first place, to carefully examine what is the actual meaning of probability" [34, p. XV]. After doing that he found that "the question what probability is remained invariably open. All attempts to answer it failed for some reason or other" [34, p. XVI]. The main reason he found is that no distinction is made between concrete objects and abstract objects [34, p. xviii] and a unified theory of probabilistic physics is developed. In that theory, he is specifically pointed out a general mistake that "it is taken for granted that quantum physics deals with the peculiarities of measuring 'observables' (that is, physical quantities) on microsystems. But there are no reasonable arguments for such a statement. The only conclusion that can be drawn from this fact is that quantum physics should deal with probabilistic treatment of microsystems, which means that it is to be based on probabilistic consideration of these systems. Besides, no true physical theory can in principle deal with peculiarities of measuring, since the measurements are always carried out on concrete objects, whereas any theory concerns abstract objects - images of the corresponding sets of concrete objects" [34, p.313]. His opinions are very much the same as ours. In the forward of the book, the German-American physicist, and philosopher of science, HENRY MARGENAU emphasized that "the key methodological principle underlying the book is of extraordinary significance and deserves special attention". He predicted that "a reader who masters all its contents will become an expert in the subject of both probability and its physical implications, while enjoying its understanding and use" [34]. We strongly recommend readers to refer to that book for a better understanding of quantum mechanics and in his opinion, "quantum mechanics and classical statistical mechanics are two interconnected domains of probabilistic physics, and at large enough quantum numbers the probability distributions for microsystems go over into the corresponding probability distributions for classical systems (macrosystems). This means that the classical limit of quantum mechanics is classical statistical mechanics, which fact has been repeatedly noted and illustrated by examples in the present book" [34, p.291].

6. Summary and Conclusions

Whether a real quantum theory is possible or not is the same as whether a theory of everything is possible or not [3,4]. These are philosophical questions of an existential statement. To refute or prove such a statement, only the observation method of all possibilities is the correct method. For the finite domain there exists a possibility, while for infinite domain, the possibility does not exist. Thus, it is basically a selection problem for the infinite domain problems. The present author has argued that to select yes is better than to select no for the advancement of science [3]. So if one starts from the very beginning by giving up these four axioms and constructs a generalization of classical mechanics to explain the quantum phenomena, the real quantum theory could be possible although complex quantum theory may have some conveniences in presentation and mathematical treatment. It is my belief that Euclidean geometry together with probability theory for handling uncertainty are adequate to explain all the phenomena in the world we can observe, no matter it is macro or micro. In the discussion of applying Newton's second law to predict the trajectory of a throwing object for five different types of objects (a stone, a coin, a cat, a person, and an electron), our conclusion is that it is not the scale that matters but the living nature whether the object is lifeless or living [33]. A scientific theory must be established on the solid philosophical foundation of clear ontology and epistemology [4]. The standard quantum theory is weak in that aspect [34], and if that belief further dominates the scientific community, the unification of classical mechanics, quantum mechanics, and relativity theory becomes impossible. This is the selection of some people rather than the proof of physical truth. Both Bohmian mechanics [35] and toy model [36] have demonstrated another two possibilities of real quantum mechanics. In the toy model, Plávala and Kleinmann have demonstrated how to construct general probabilistic theories that contain an energy observable dependent on position and momentum. The construction is in accordance with classical and quantum theory and allows for physical predictions, such as the probability distribution for position, momentum, and energy. Similar to the idea of toy model, I select general system theory as a tool to start with, since every problem we encounter can be modelled by a system, and if a unique set of axioms is used to construct the theory, then the construction of theory of everything for the world we can observe is possible but certainly not for the whole universe [3,4,33]. Similar as the emphasis made by Mayants to distinguish the concrete objects and the abstract objects, we emphasize the distinction between world (finite spacetime) and universe (infinite spacetime).

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